Chapter 5. Evaluation of Functions

5.0 Introduction

The purpose of this chapter is to acquaint you with a selection of the techniques that are frequently used in evaluating functions. In Chapter 6, we will apply and illustrate these techniques by giving routines for a variety of specific functions. The purposes of this chapter and the next are thus mostly in harmony, but there is nevertheless some tension between them: Routines that are clearest and most illustrative of the general techniques of this chapter are not always the methods of choice for a particular special function. By comparing this chapter to the next one, you should get some idea of the balance between "general" and "special" methods that occurs in practice.

Insofar as that balance favors general methods, this chapter should give you ideas about how to write your own routine for the evaluation of a function which, while "special" to you, is not so special as to be included in Chapter 6 or the standard program libraries.

CITED REFERENCES AND FURTHER READING:

Fike, C.T. 1968, *Computer Evaluation of Mathematical Functions* (Englewood Cliffs, NJ: Prentice-Hall).

Lanczos, C. 1956, *Applied Analysis*; reprinted 1988 (New York: Dover), Chapter 7.

5.1 Series and Their Convergence

Everybody knows that an analytic function can be expanded in the neighborhood of a point x_0 in a power series,

$$f(x) = \sum_{k=0}^{\infty} a_k (x - x_0)^k \tag{5.1.1}$$

Such series are straightforward to evaluate. You don't, of course, evaluate the kth power of $x - x_0$ *ab initio* for each term; rather you keep the $k-1$st power and update it with a multiply. Similarly, the form of the coefficients a is often such as to make use of previous work: Terms like $k!$ or $(2k)!$ can be updated in a multiply or two.