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# Distributed(Direct ) Interconnection Networks

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AMANO, Hideharu

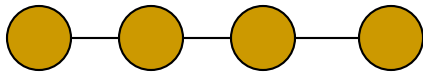
Textbook pp.140—147

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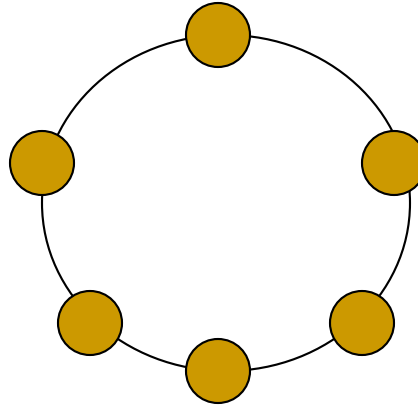
# Distributed (Direct Interconnection ) Networks

- Nodes are connected with links directly.
  - Locality of communication can be used.
  - Extension to large size is easy.
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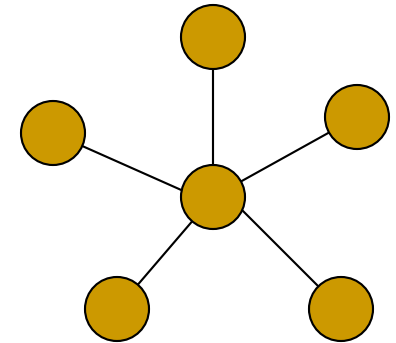
# Basic direct networks



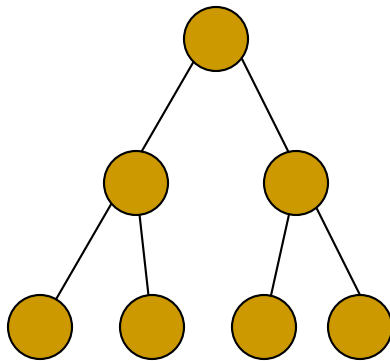
Linear



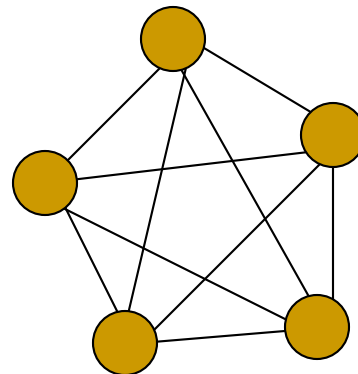
Ring



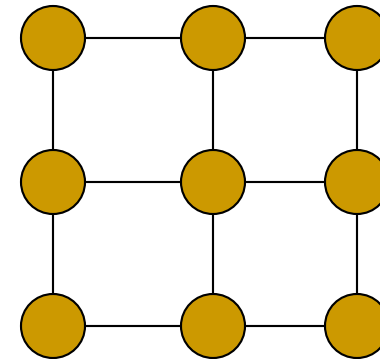
Central concentration



Tree



Complete connection



Mesh

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# Metrics of Direct interconnection network (D and d)

- Diameter: D
  - Number of hops between most distant two nodes through the minimal path
- degree: d
  - The largest number of links per a node.
- D represents performance and d represents cost

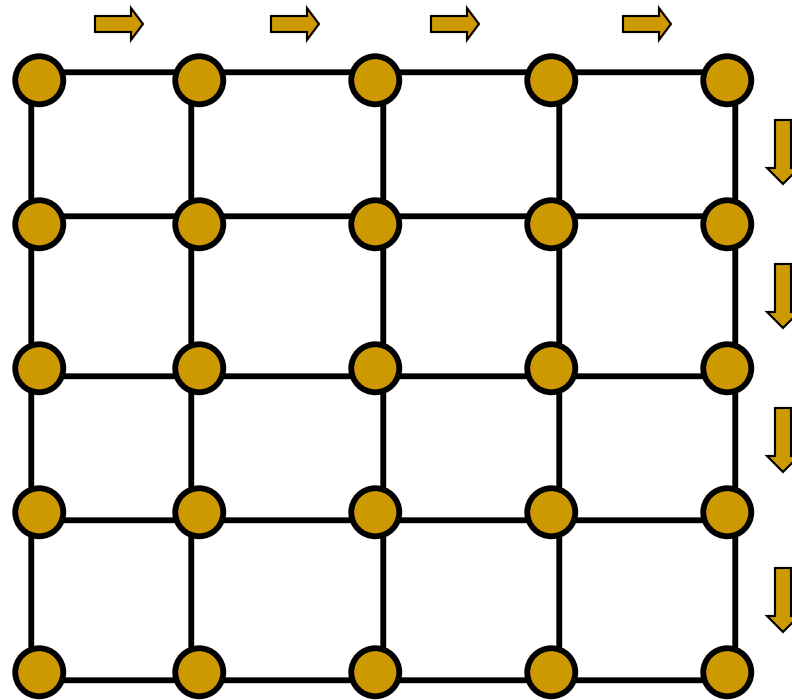
Recent trends:

Performance: Throughput

Cost: The number of long links

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# Diameter



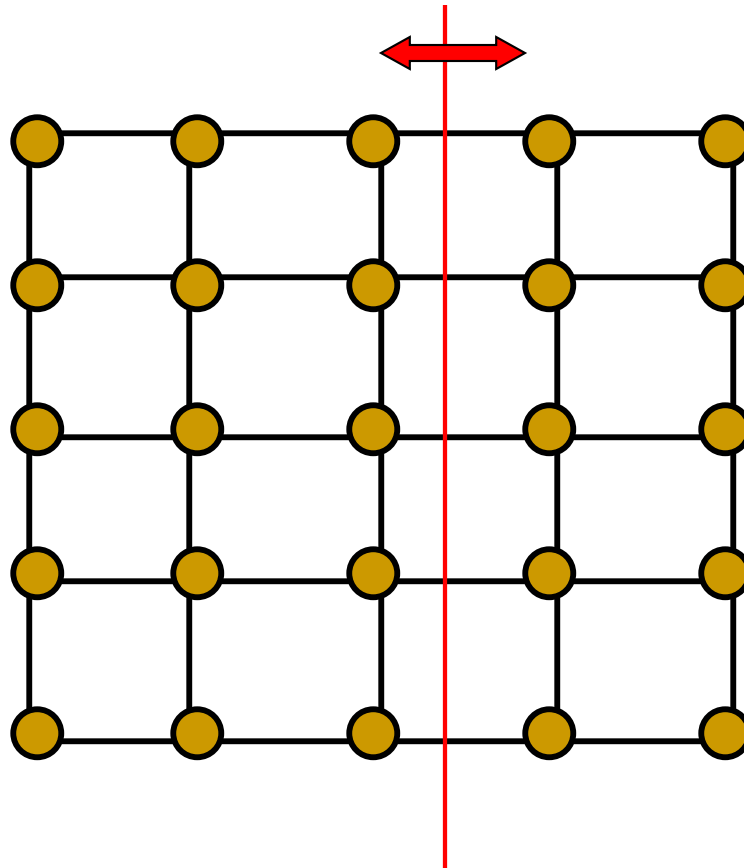
$$2(n-1)$$

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# Other requirements

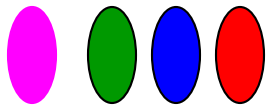
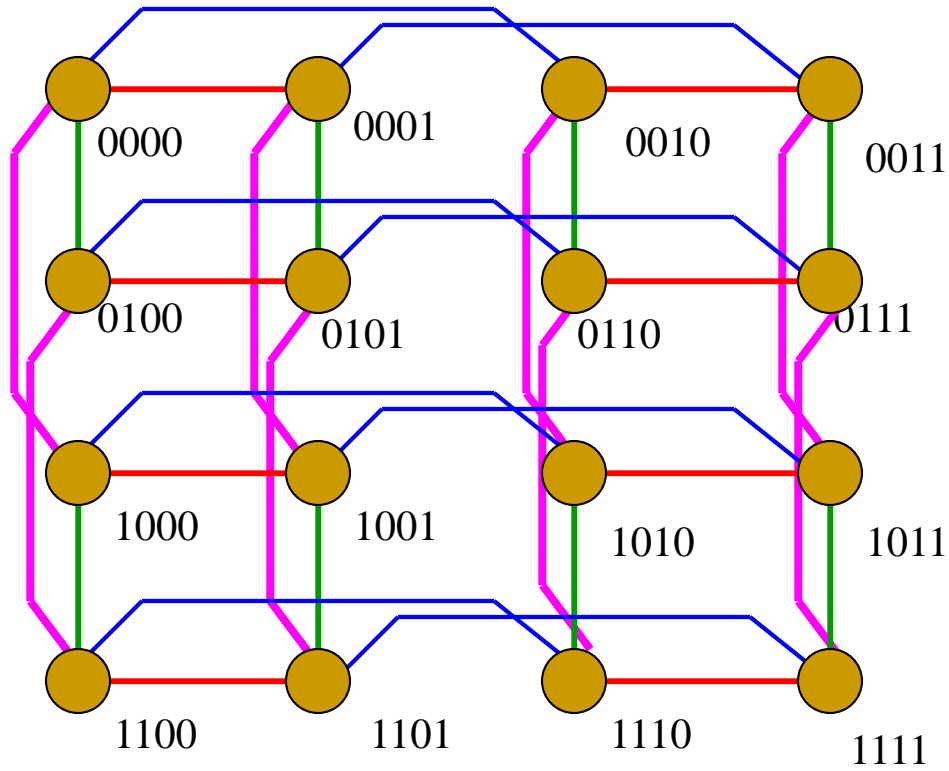
- Uniformity: Every node/link has the same configuration.
  - Expandability: The size can be easily extended.
  - Fault Tolerance: A single fault on link or node does not cause a fatal damage on the total network.
  - Embeddability: Emulating other networks
  - Bisection Bandwidth
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# bi-section bandwidth



The total amount of data traffic between two halves of the network.

# Hypercube

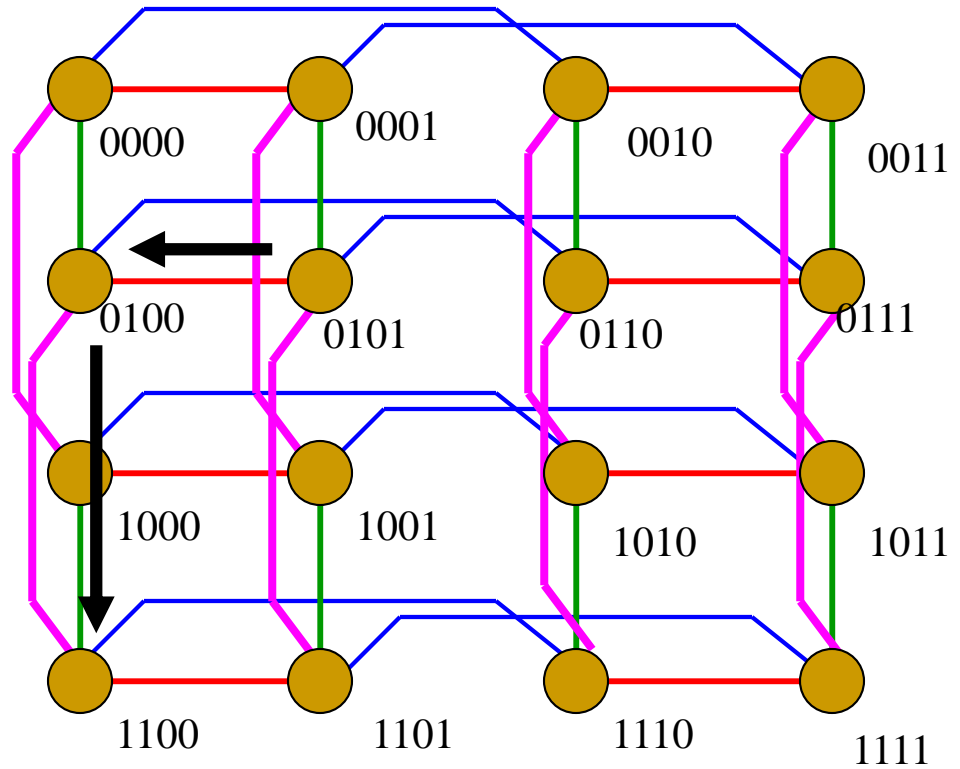
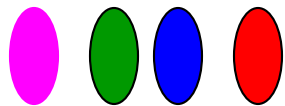




# Routing on hypercube

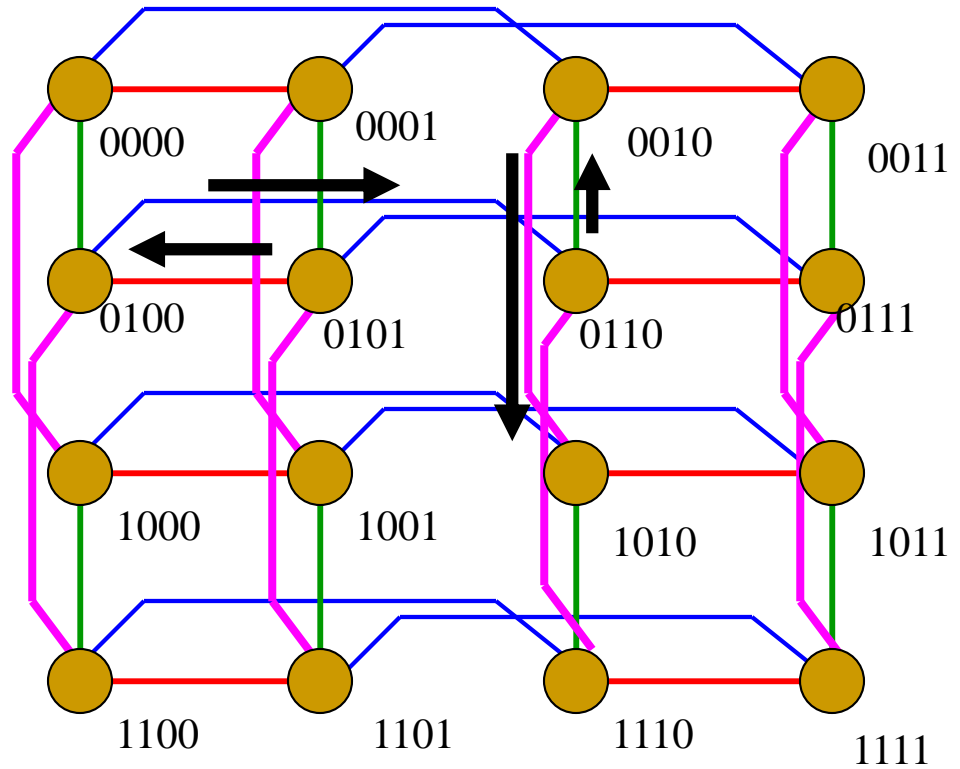
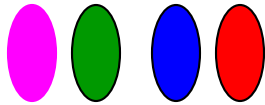
0101 → 1100

Different bits



# The diameter of hypercube

$0101 \rightarrow 1010$   
All bits are different  
 $\rightarrow$  the largest distance



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# Characteristics of hypercube

- $D=d=\log N$
  - High throughput, Bisection Bandwidth
  - Embeddability for various networks
  - Satisfies all fundamental characteristics of direct networks (Expandability is questionable)
  - Most of the first generation of NORA machines are hypercubes (iPSC, NCUBE, FPS-T)
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# Problems of hypercube

- Large number of links
    - Large number of distant links
    - High bandwidth links are difficult for a high performance processors.
  - Small  $D$  does not contribute performance because of innovation of packet transfer.
  - Programming is difficult: → Hypercube's dilemma
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# Is hypercube extendable?

- Yes (Theoretical viewpoint)
  - The throughput increases relational to the system size.
- No (Practical viewpoint)
  - The system size is limited by the link of node.



# Hypercube's dilemma

- Programming considering the topology is difficult unlike 2-D,3-D mesh/torus
- Programming for random communication network cannot make the use of locality of communication.



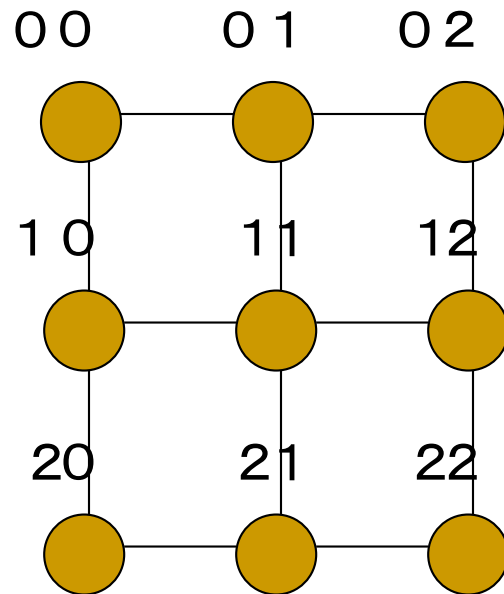
- 2-D/3-D mesh/torus
  - Killer applications fit to the topology
    - Partial differential equation, Image processing,...
  - Simple mapping strategies
    - Frequent communicating processes should be Assigned to neighboring nodes

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# k-ary n-cube

- Generalized mesh/torus
  - K-ary n digits number is assigned into each node
  - For each dimension (digit), links are provided to nodes whose number are the same except the dimension in order.
  - Rap-around links ( $n-1 \rightarrow 0$ ) form a torus, otherwise mesh.
  - “high-n” networks are used in recent supercomputers
    - Tofu in K uses 6-torus
    - Bluegene Q uses 5-torus
-

# k-ary n-cube

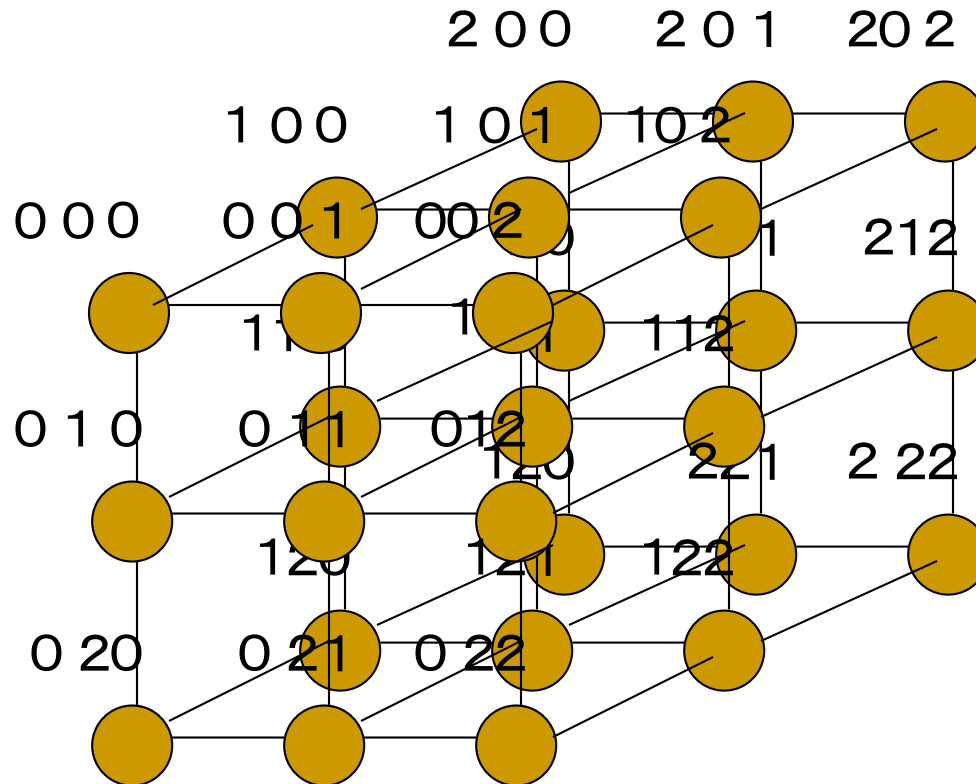


3-ary 1-cube

3-ary 2-cube



# k-ary n-cube

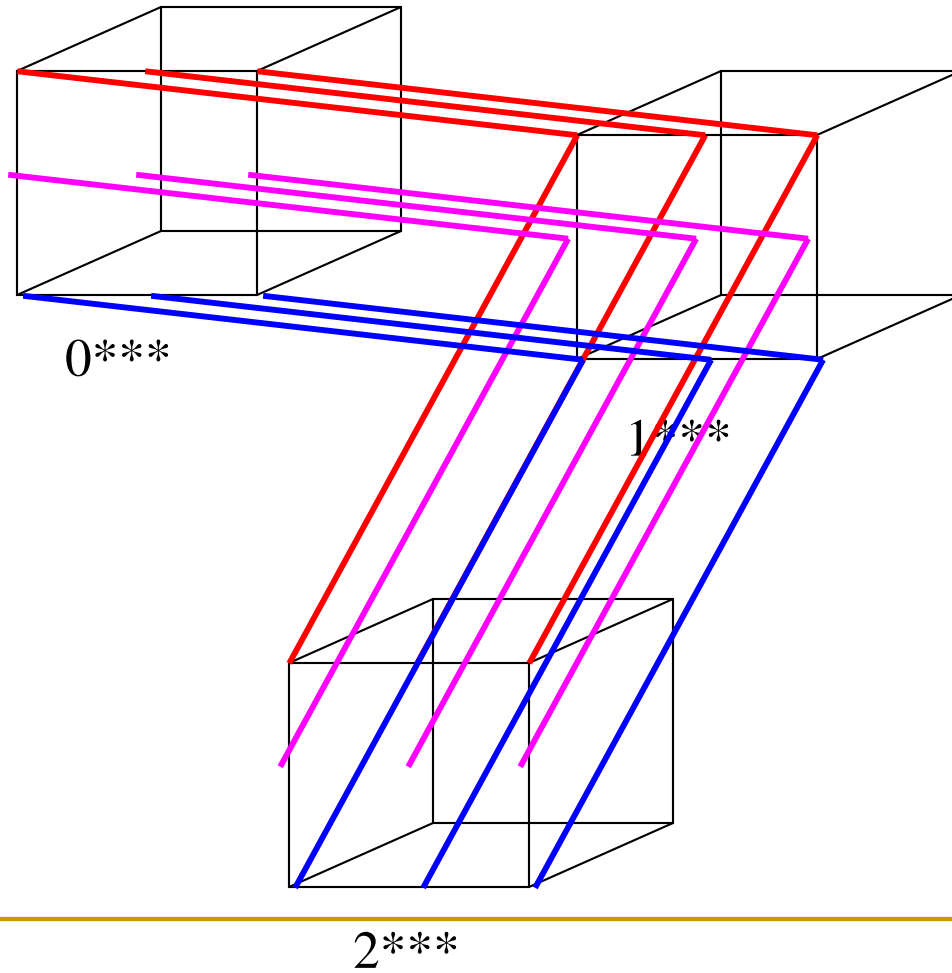


3-ary 1-cube

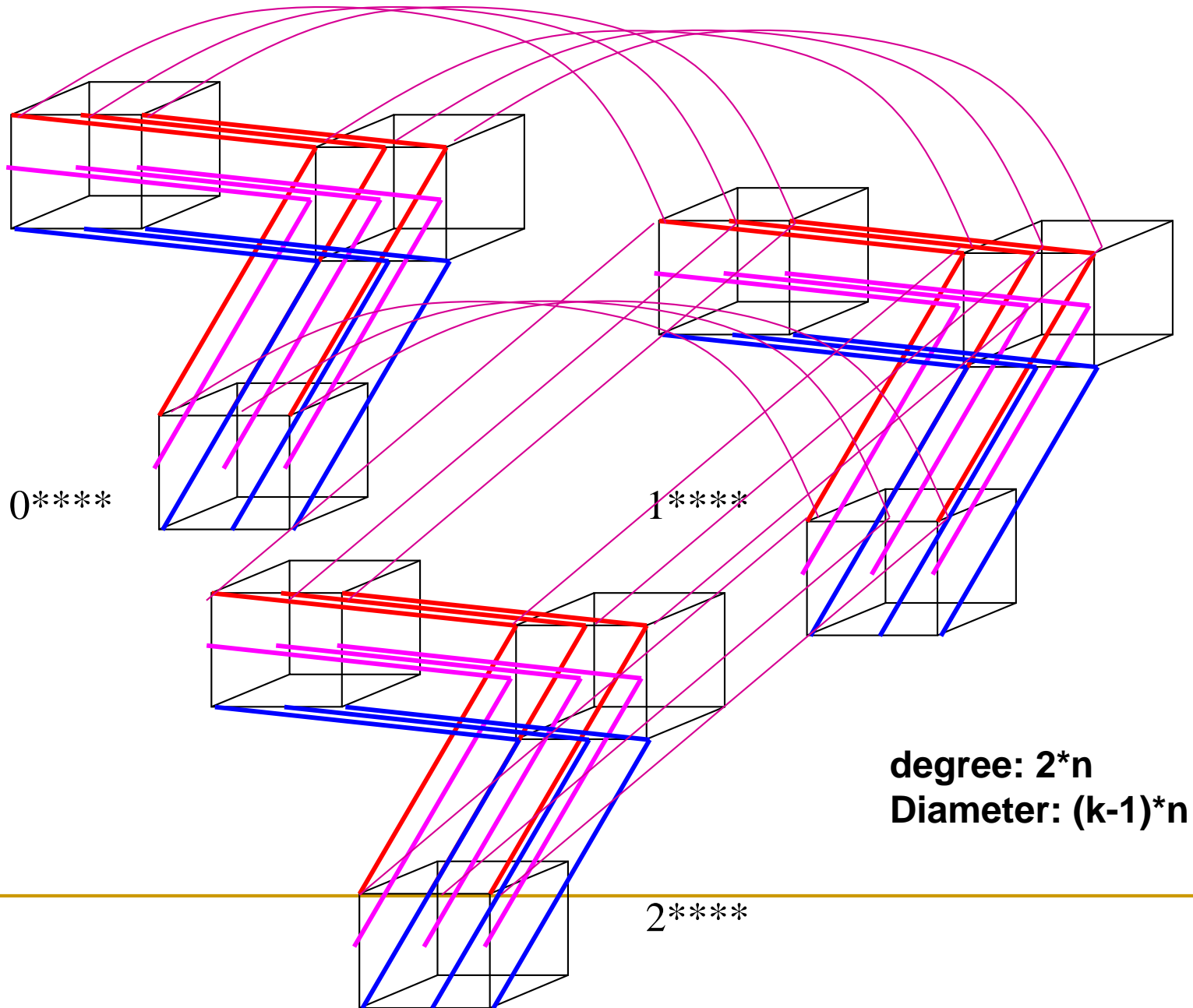
3-ary 2-cube

3-ary 3-cube

# 3-ary 4-cube



# 3-ary 5-cube



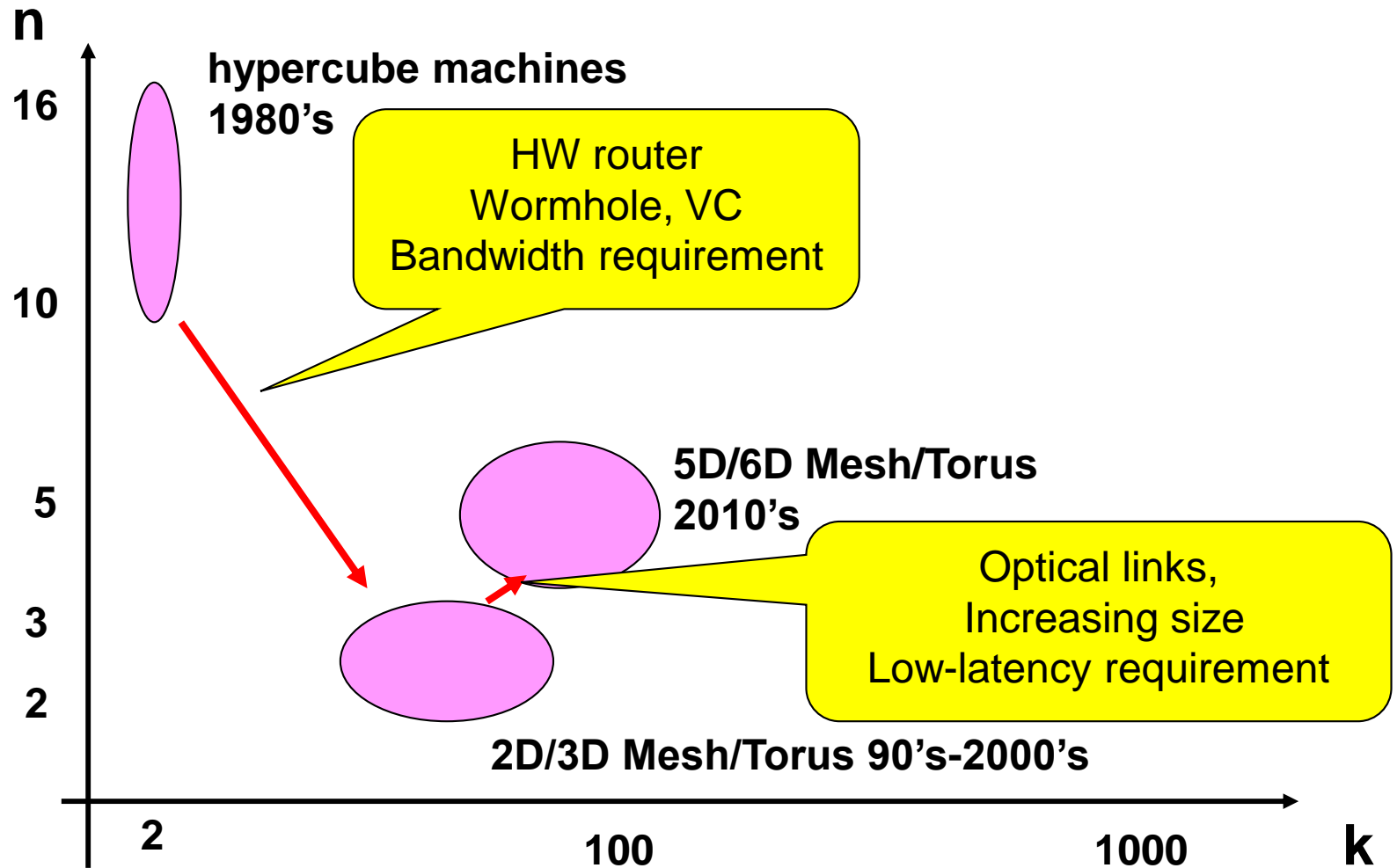


**6-dimensional Torus  
Tofu**

# Properties of k-ary n-cube

- A class of networks which has Linear, Ring 2-D/3-D mesh/torus and Hypercube (binary n-cube) as its member.
- Small  $d=2n$  but large  $D(O(k^{1/n}))$
- Large number of neighboring links
- k-ary n-cube has been a main stream of NORA networks. Recently, small-n large-k networks are trendy.

# Rise and fall of the members



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# Quiz

- Calculate Diameter ( $D$ ) and degree ( $d$ ) of the 6-ary 4-cube (mesh-type).
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# Glossary 1

- Diameter:直径
- degree:次数
- Uniformity:均一性
- Expandability:拡張性
- Embeddability:埋め込み能力
- Bisection bandwidth:2分割間転送量
- Torus:両端が接続されたネットワークで、特にメッシュに対するものを指す。複数形はToriなので注意
- n-ary k-cube: n進kキューブ 2進キューブのことを特にハイパーキューブと呼ぶ

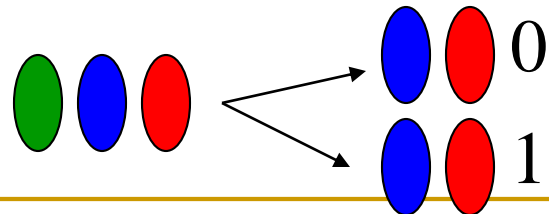
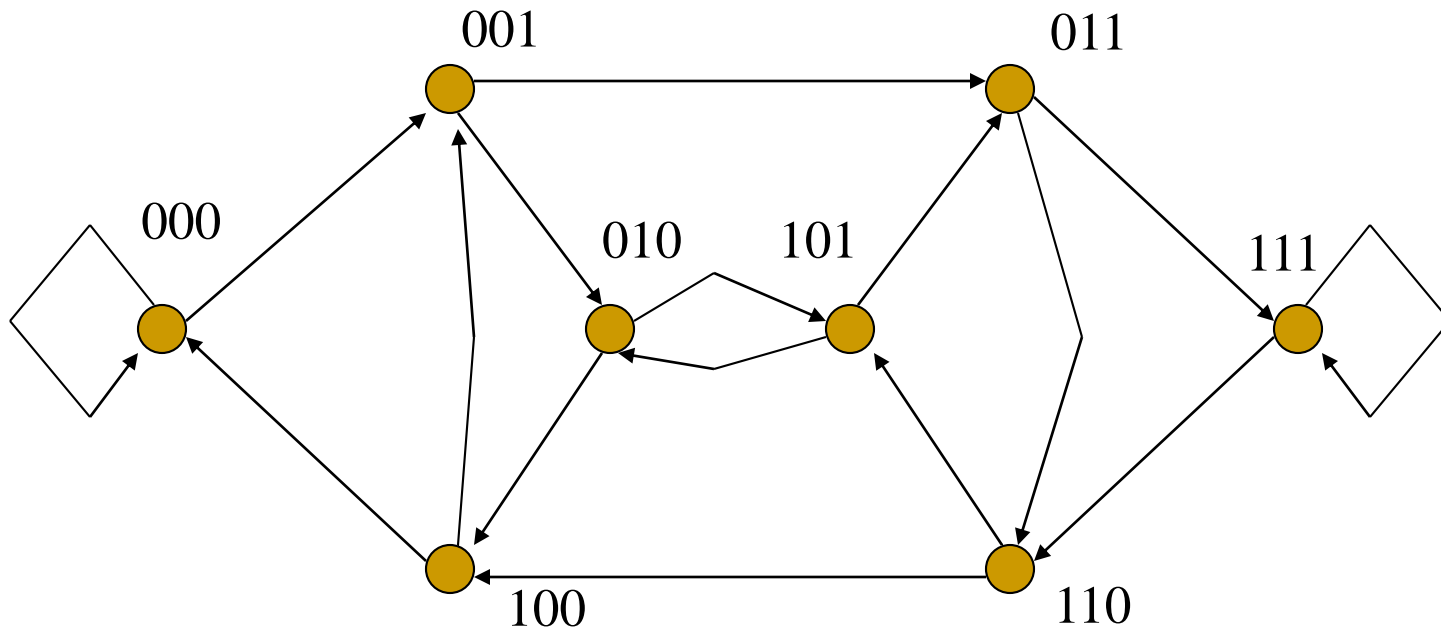


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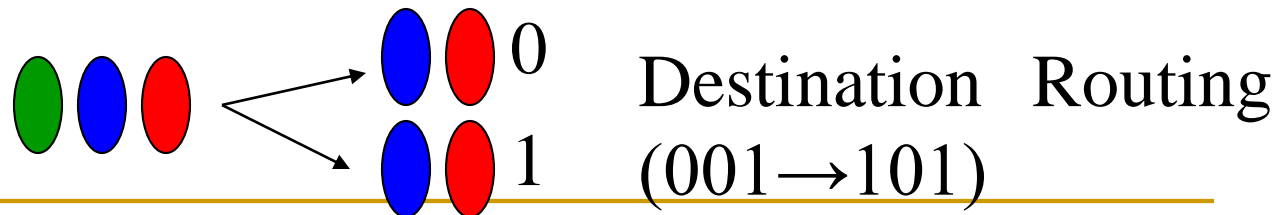
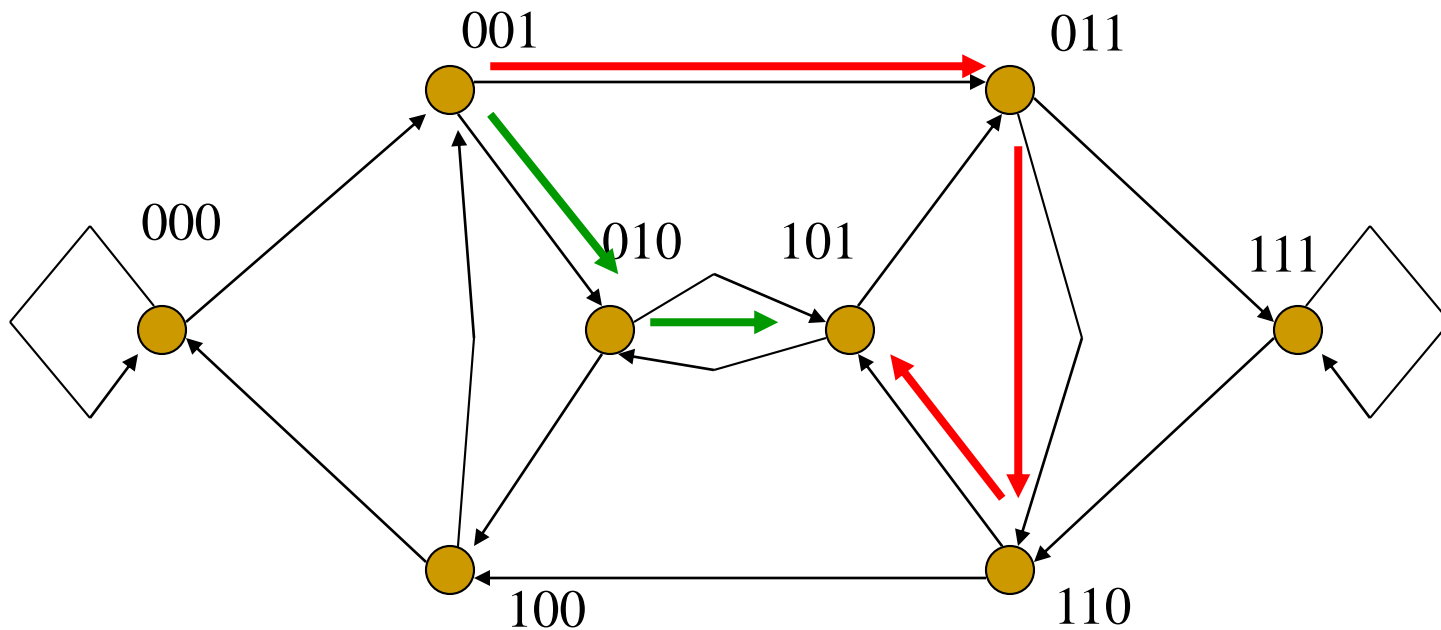
# Advanced direct networks

- Shuffle based networks
    - De Bruijn, Kautz, Pradhan
  - Extended mesh/torus
    - Midimew, RDT
  - Star Graph
  - Hierarchical networks
    - CCC, Hypernet
  - Circular networks
    - Circular Omega, MDCE
  - Network inside the chip (Network-on-Chip)
    - Spidergon, Mesh of Tree, Fat-H Tree
    - Some of them might be classified into indirect networks
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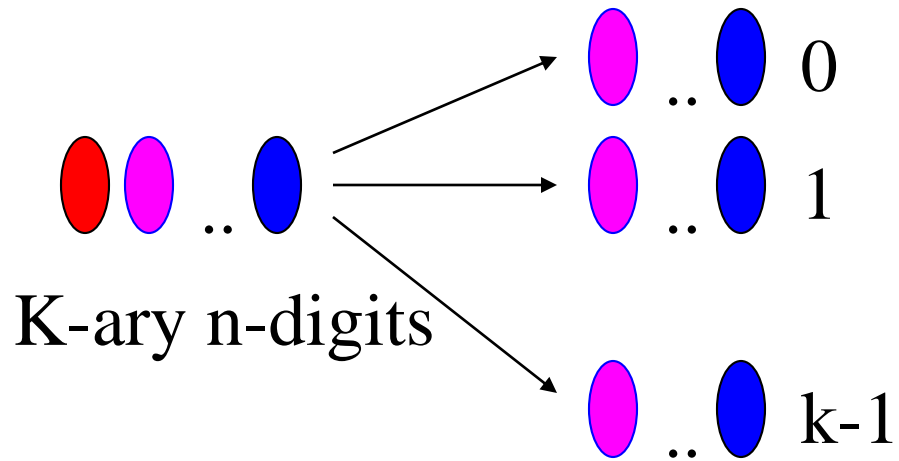
# De Bruijn network



# Routings for De Bruijn



# $B(k, n)$



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# Characteristics of De Bruijn

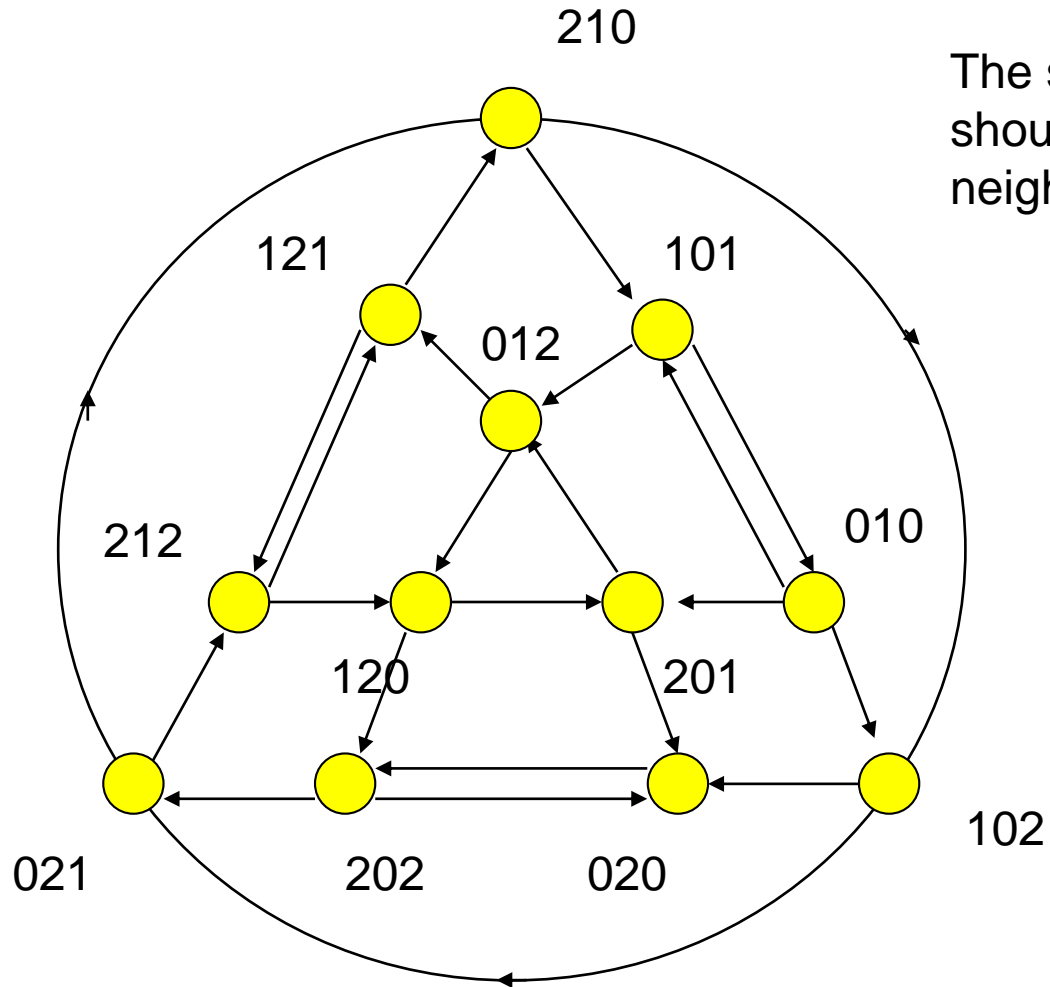
## ■ Benefits

- $d=2k$ ,  $D=n=\log N$
- When  $k=2$ ,  $d=4$ ,  $D=\log N$ , that is,  $d$  of 2-dimensional mesh but  $D$  of hypercube.

## ■ Problems

- Optimal routing is difficult (not established yet).
  - Destination routing cannot make a best use of communication locality.
  - No killer applications.
  - Self loop and duplicated links
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# Kautz network



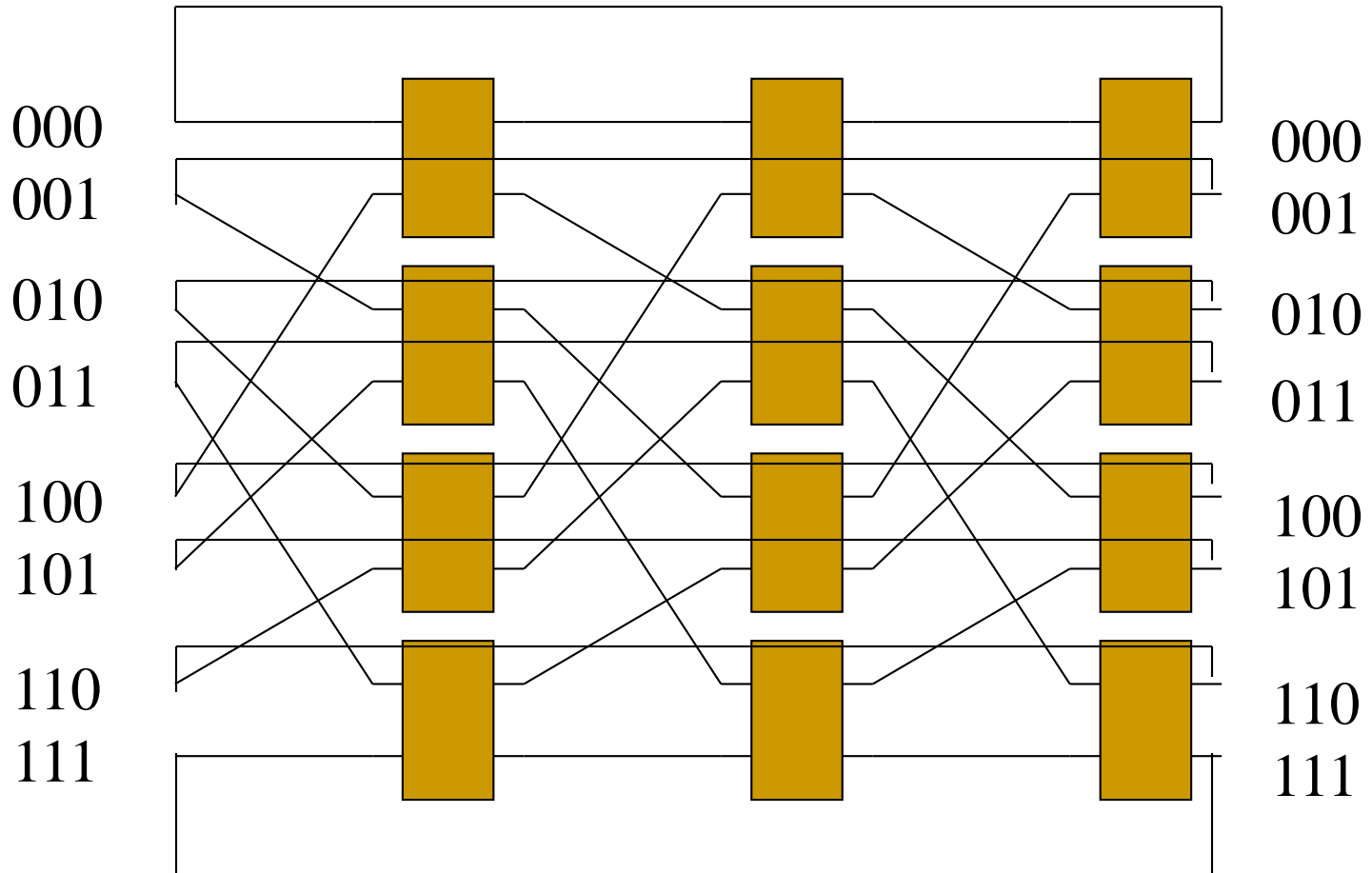
The same number should not be at the neighboring digit

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# Circular networks

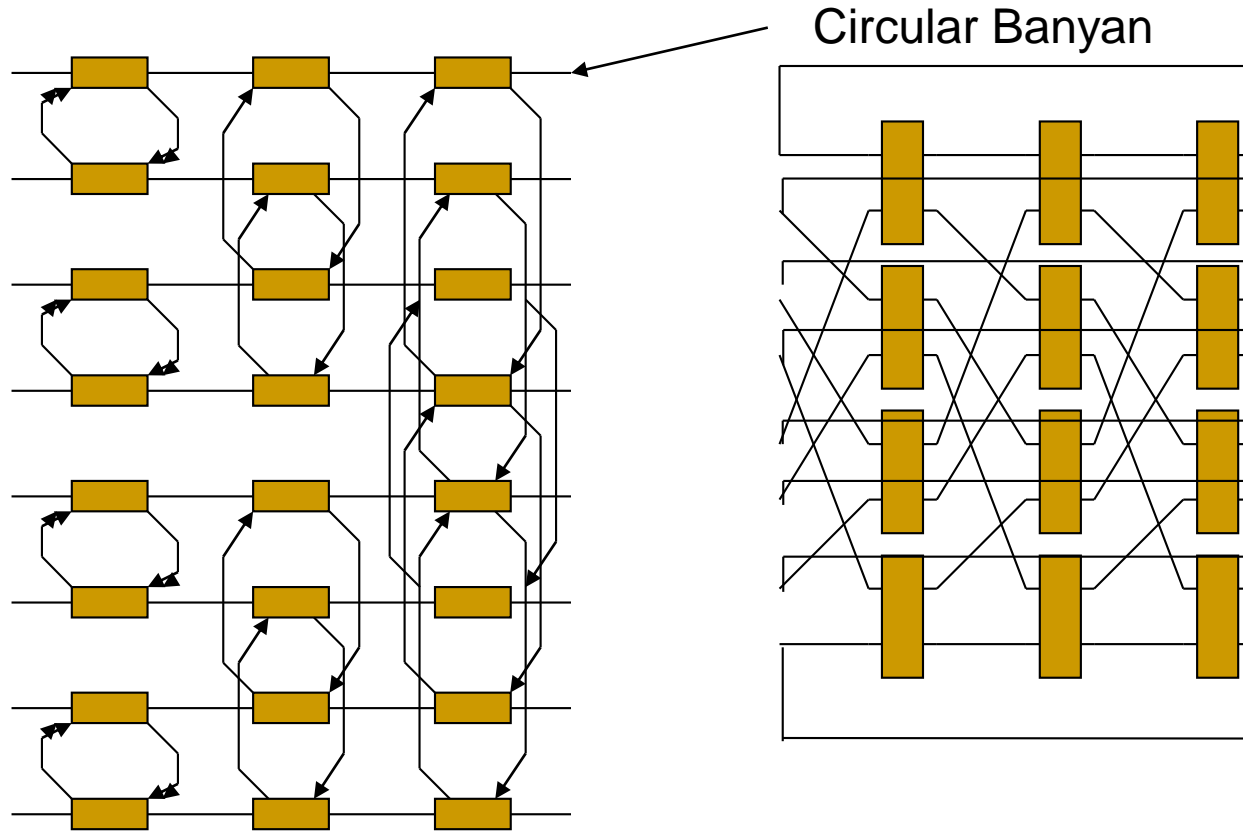
- Circular Omega
    - Advantageous for one-way communication
    - Used in data-flow machine EM-4
  - MDCE(CCCB)
    - Hierarchical structure of Circular Omega (Banyan)
    - Used in massively parallel machine RWC-1
-

# Circular Omega network





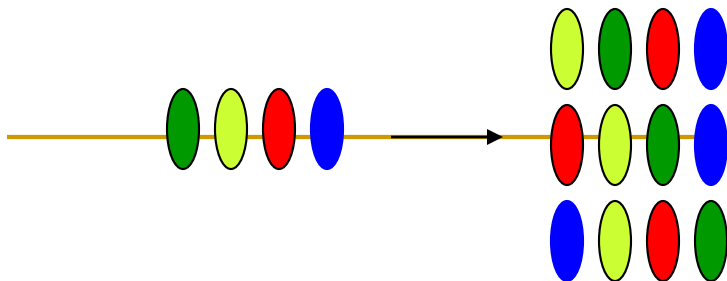
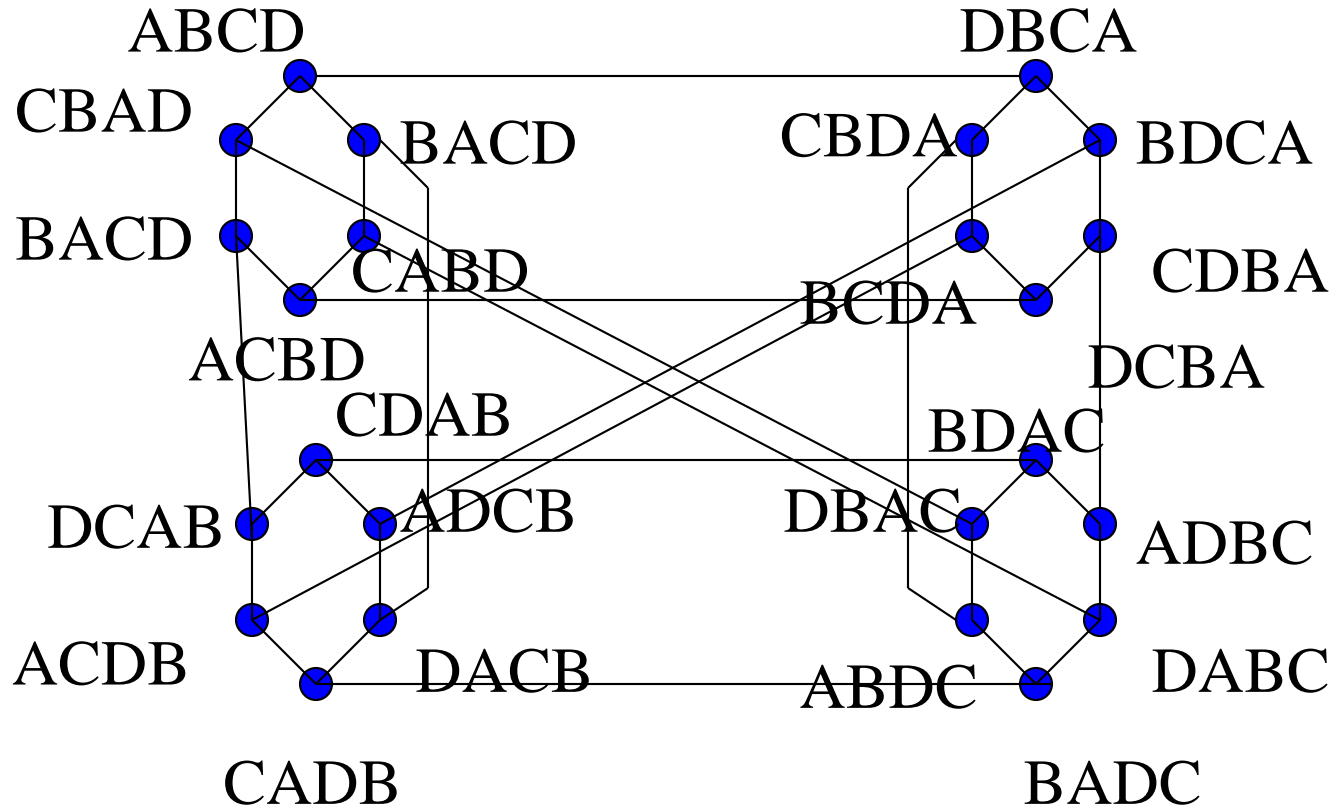
# Cube Connected Circular Banyan



3-Dimensional

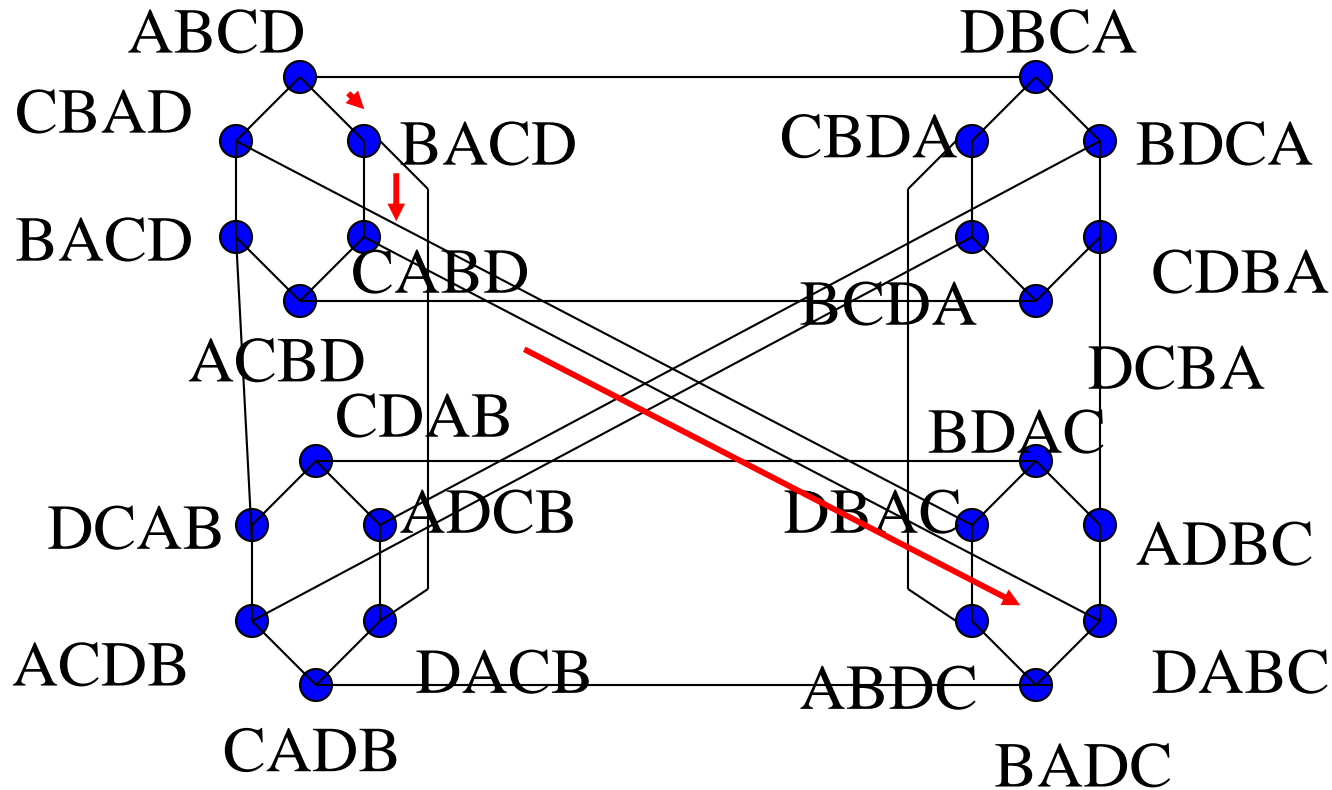
Proposed for RWC-1

# Star graph



Connection  $n!$  nodes

# Routing on Star graph



If A is top, change with arbitrary symbol,  
 else, change with the symbol of destination

ABCD → DABC

node

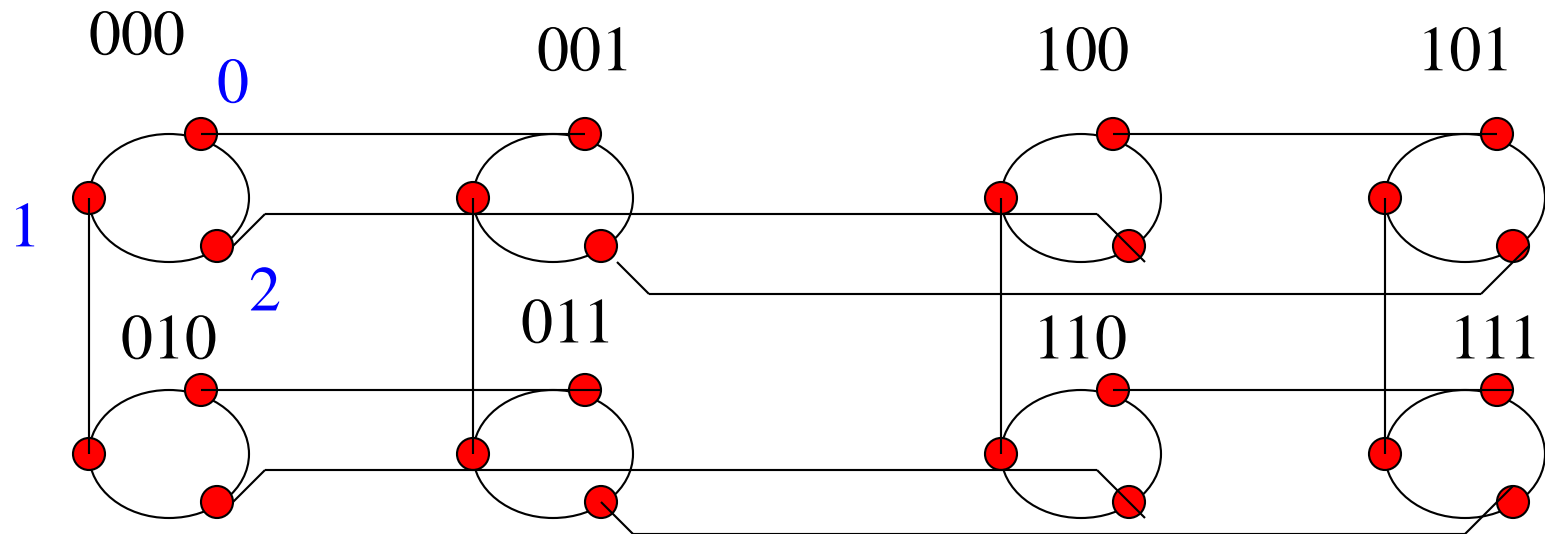
$$3(n-1)/2$$

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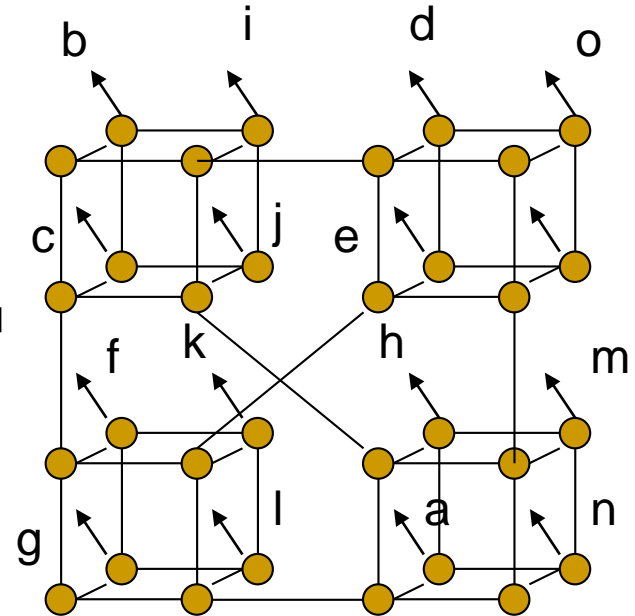
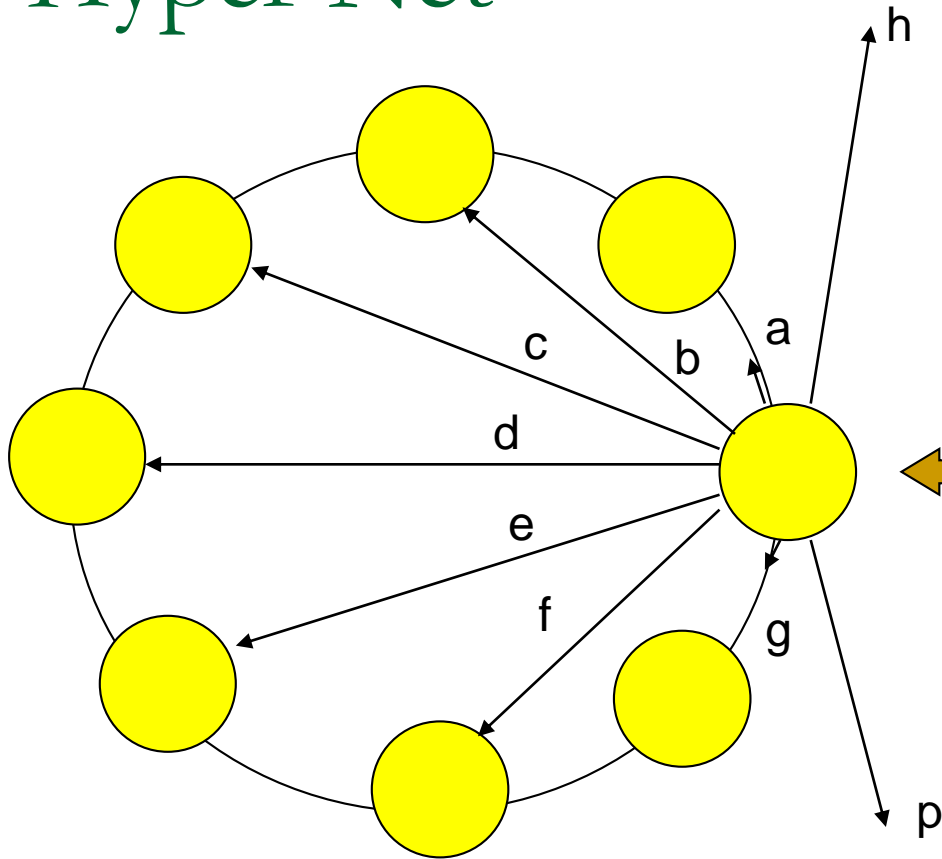
# Hierarchical network

- CCC(Cube Connected Cycles)
    - hypercube + loop
  - Hypernet
    - Complete connection + hypercube
  - Well combined, weak points of original networks are vanished.
  - Complicated routing, gap between hierarchies
-

# CCC(Cube Connected Cycles)



# Hyper Net



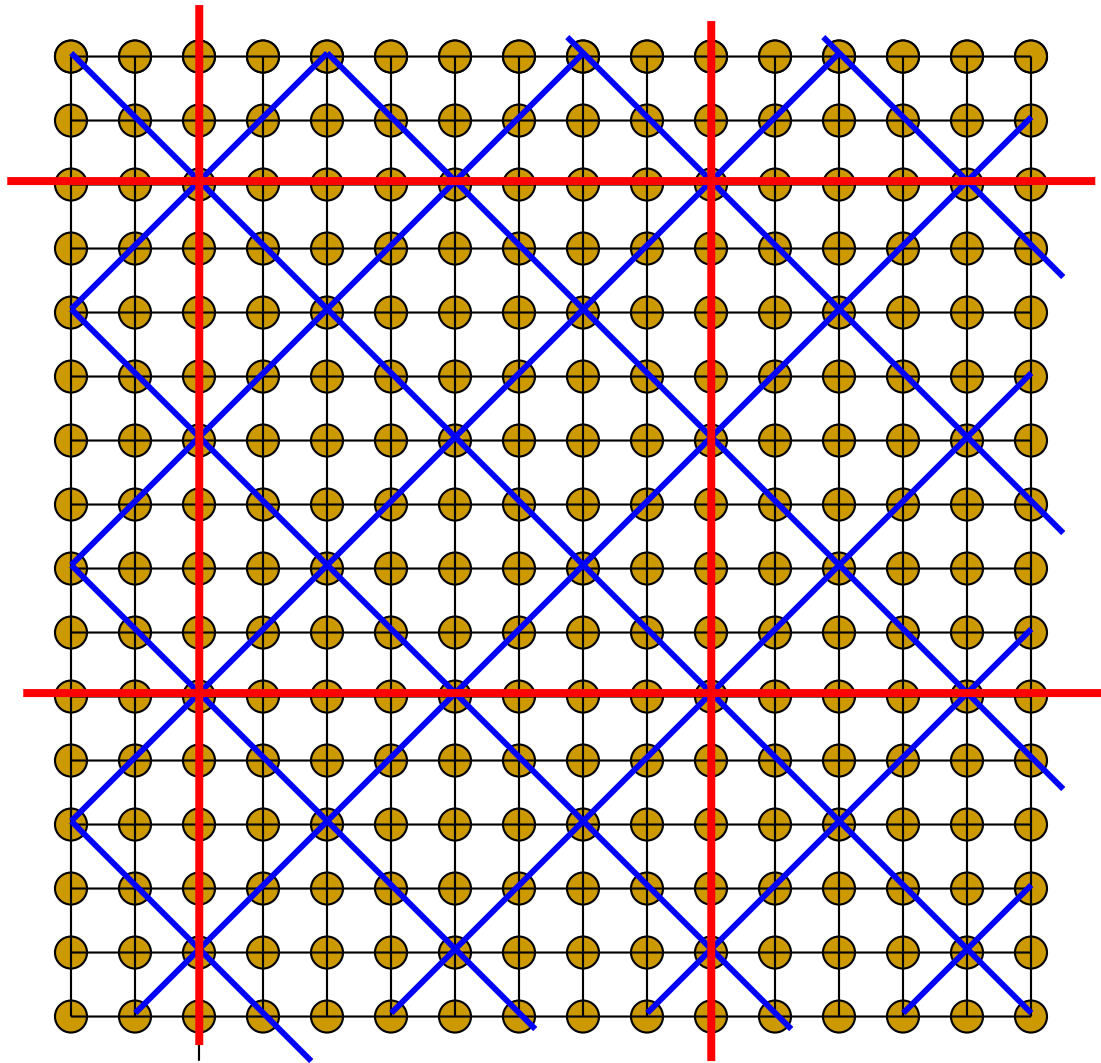
Other links are used for further upper hierarchy

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# Extended mesh/torus

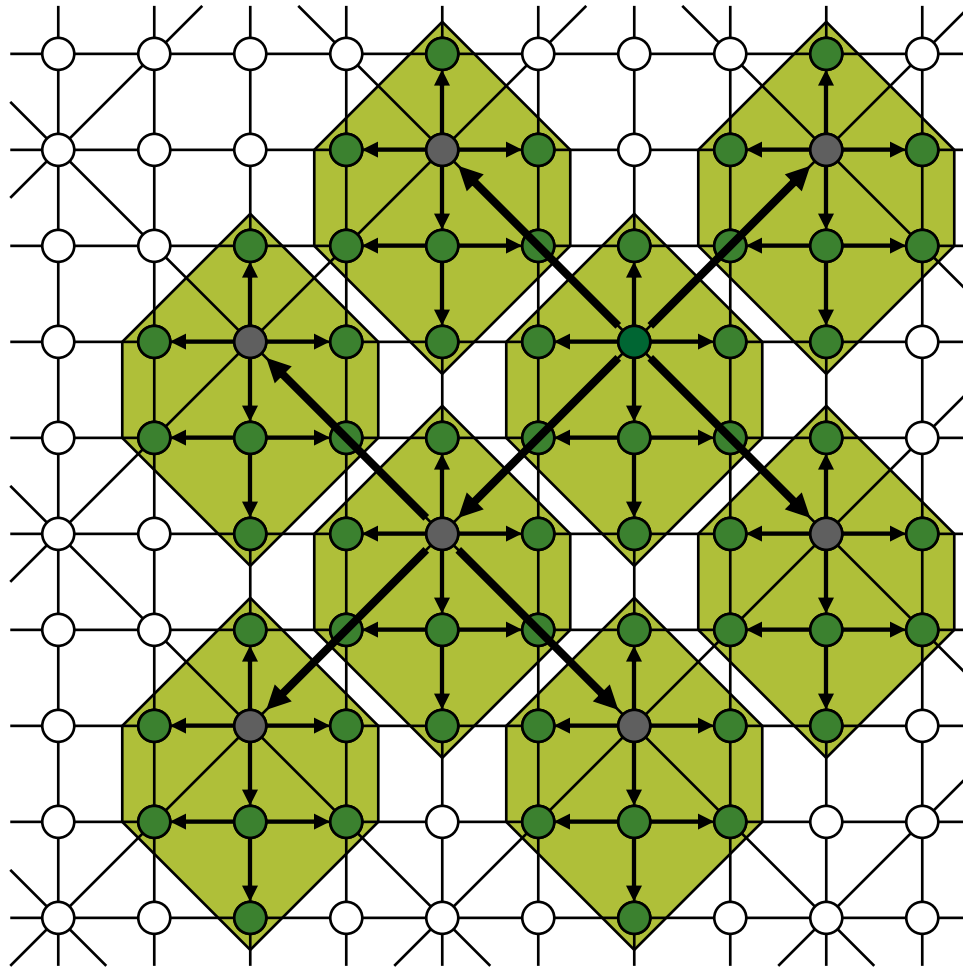
- Including mesh/torus structure
  - Extended links for performance enhancement
    - Reconfigurable Mesh
    - Midimew
    - RDT
-

# RDT(Recursive Diagonal Torus)





# Multicasting on the RDT



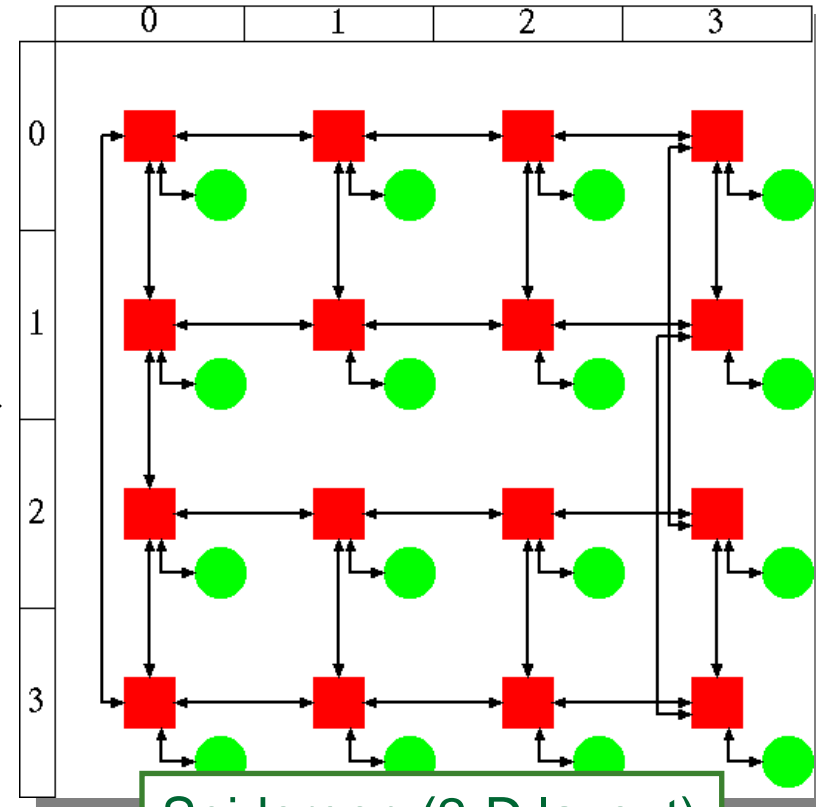
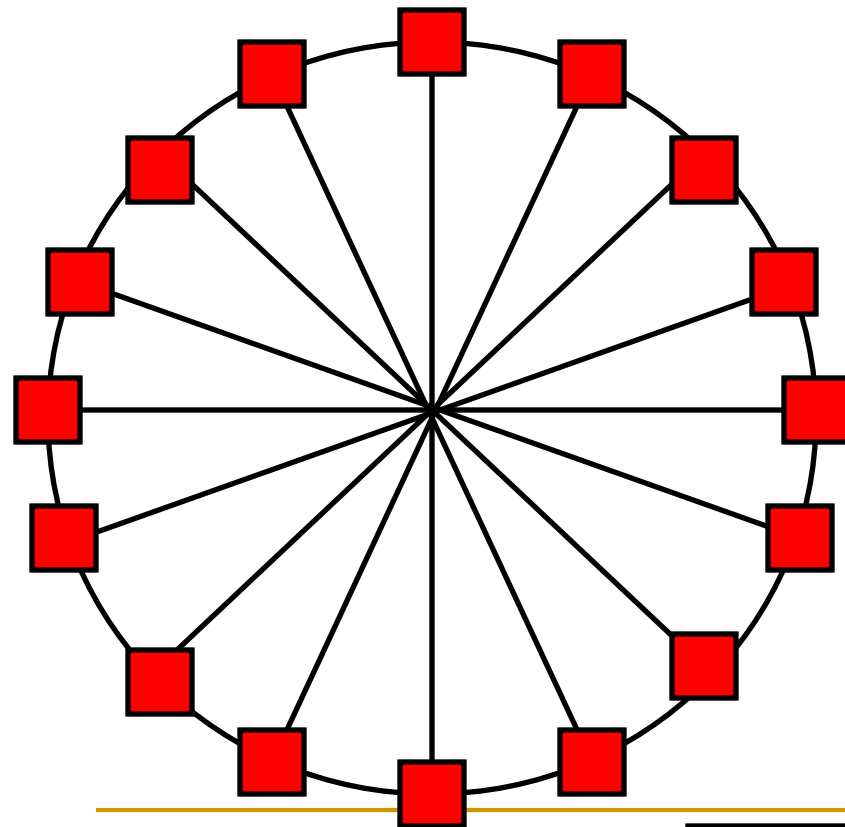
# Topology for NoC: (1)

## ■ Spidergon

- Ring + diagonal links
- Node degree 3;

[Coppola, ISSOC'04]

[Bononi, DATE'06]



Spidergon (2-D layout)

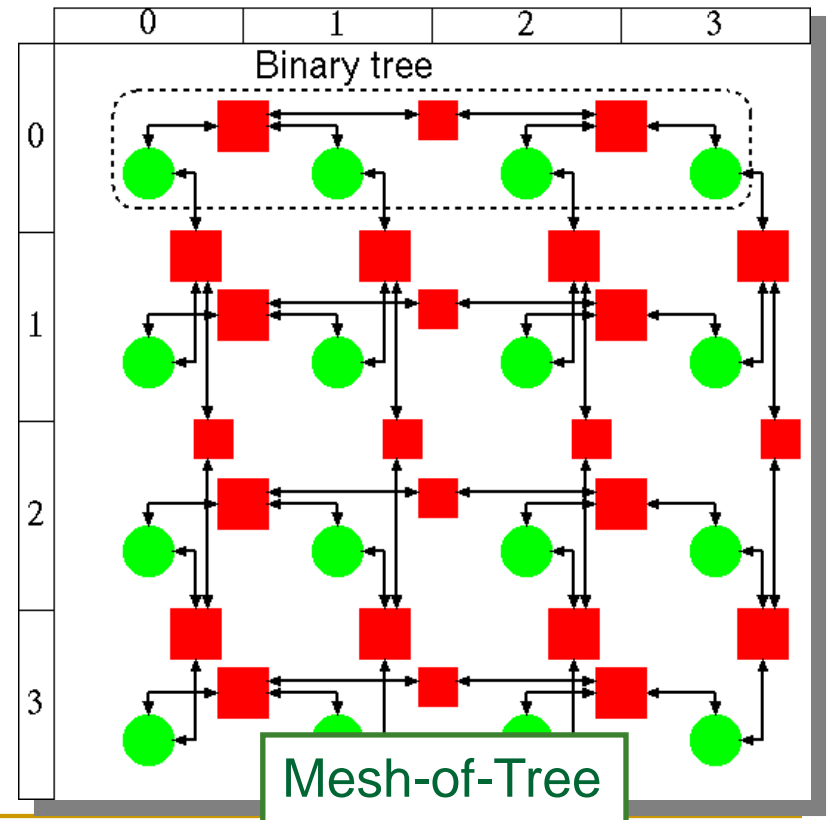
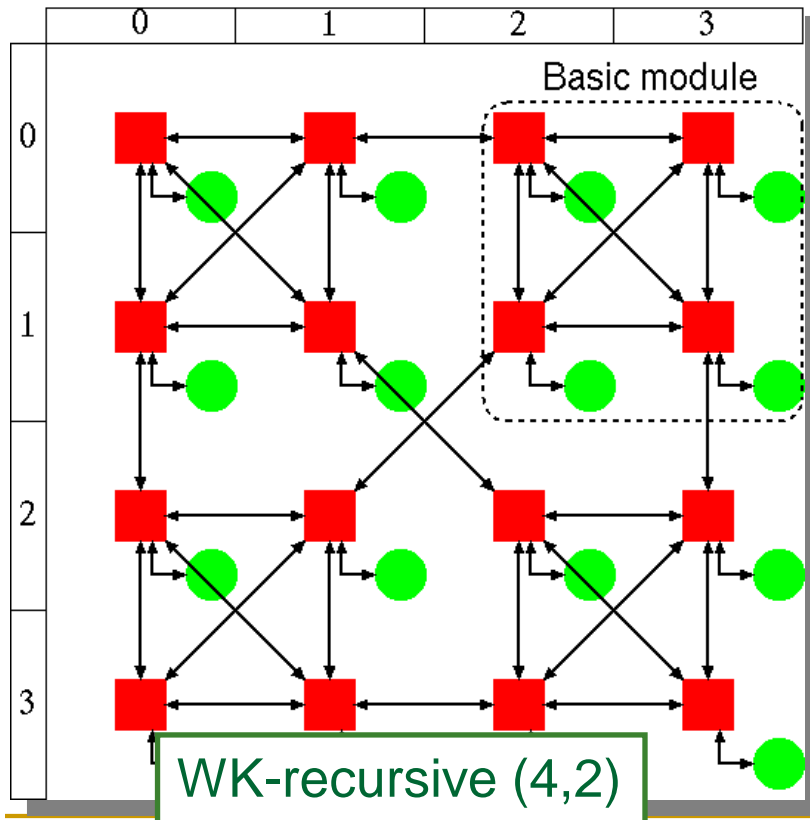
■ router

● core

# Topology for NoC: (2)

- WK-recursive ( $d, k$ )
  - *hierarchical network*

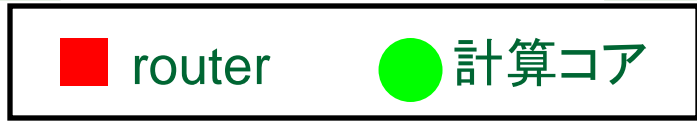
- Mesh-of-Tree
  - Mesh + Tree



[Vecchia, FCGS'88]

[Rahmati,

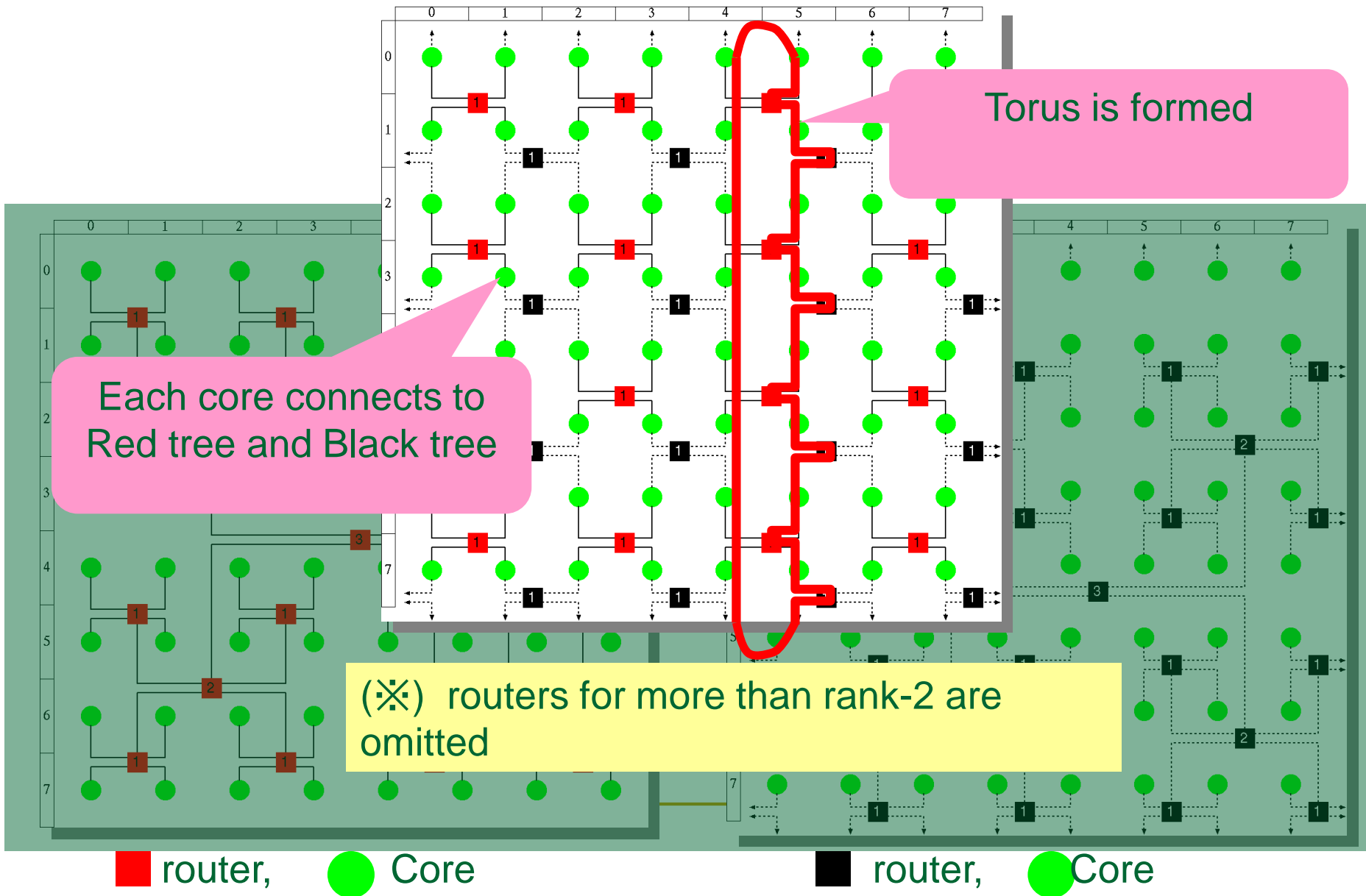
ICCD'00]



[Leighton, Math System

1991]

# Fat H-Tree: A network topology for NoCs



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# Glossary 2

- De Bruijin: 人の名前でドブロイアンと読むのが本来の読み方だが英語圏の人はこれをデブルージンと読むので注意（最初全然わかんなかった）
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# Summary

- Recently, practical new topologies are not proposed.
  - A lot of “made-in-Japan” networks
  - Asymmetric indirect networks will be widely used.
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# Exercise

- Compute diameter of CCC with 16 cycles each of which has 4 nodes.

Hint: How is the method to move between cycles efficiently?

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